CS673-F20-Homework-Template (Change this to Homework 01, e.g.)

Your Name goes here (Please write your full name here)

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Instructions: Please answer the following questions to the best of your ability.

- Please read each problem's statement carefully to determine what you need to write in your answer.
- **Cite any sources you reference** including textbook, whatever you find on the Internet, etc. A failure to cite the sources can result in an F in the course grade as it violates the Honor Code.
- If you have any questions about LaTeX, please search the Internet (using your favorite search engine!) before asking on Piazza.

1 Answer to Problem 1

Write your answer to Problem 1 here.

You will find this quick introduction to LaTeX useful: A link to a YouTube video (this text is a hyperlink) .

As an example, you can use the in-line math mode as follows: $\forall x \in \mathbf{N}$, $x < x + 1$ always holds.

For a longer equation, you can do something like this (and give it a unique label):

$$\sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2} \tag{1}$$

You can refer to your equation like this: Equation 1. In addition, a series of inequalities or equations could be useful. Let's prove the formula above using induction.

Statement: $\sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2}$ for all integers $n \geq 1$.

Base case: When $n = 1$, the equation trivially holds.

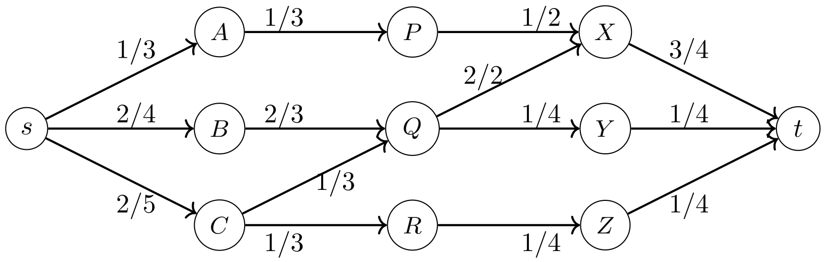
Inductive Hypothesis: Suppose that Equation 1 is true for some integer $k \geq 1$. We will prove for the case $n = k + 1$. By expanding the summation, we can re-write the left-hand-side as follows:

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \left(\sum_{i=1}^{k} i\right) + (k+1) && (2) \\
&= \frac{k \cdot (k+1)}{2} + (k+1) && (3) \\
&= \frac{(k+1)(k+2)}{2} && (4)
\end{aligned}$$

Notice that from Equation 2 to Equation 3, we are applying the Inductive Hypothesis.

If you want to draw a diagram or a graph, using `tikz` package would be the best. Here is an example:

You can also include an image (PNG file is preferred) using the `includegraphics` command. Search the Internet to learn how to use it.



2 Answer to Problem 2

Let's say you want to include pseudocode as a part of your answer. Note that it's often useful to mix your code and explanations to concisely describe what your algorithm is supposed to do.

`Dummy`(l, r) // Invariant: $0 \leq l < r \leq n$.

$m \leftarrow$`RANDOM`(l, r) // Returns an integer from $[l, r)$, uniformly at random.
$v \leftarrow m^2$
$w \leftarrow \lfloor m/2 \rfloor$
if $v < w$ **then**
  return "Whaaat"
else if $x = v$ **then**
  return "Huh"
else
  return "Of Course!"
end if

Sometimes you will be asked to prove a statement. You can use the Theorem-proof environment like below.

Theorem 1. *For any natural number $n \in \mathbf{N}$ (in other words, $n \geq 1$ and $n \in \mathbf{Z}$, if n^2 is an even number, then n must be an even number as well.*

Proof. We will prove the contrapositive of the statement: If n is NOT an even number (that is, n is an odd number), then n^2 must NOT be an even number (that is, n^2 must be an odd number).

Let n be an odd number, which can be written as $n = 2k + 1$ for some integer $k \geq 0$. Then, $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2 \cdot (2k^2 + 2k) + 1$, which implies that n^2 is also an odd number. □

Here is another way to prove the same theorem using contradiction.

Alternative proof. Suppose that n^2 is an even number but n is an odd number (which would lead to a contradiction). Since n is an odd number, we can write it as $n = 2k + 1$ for some integer $k \geq 0$. Then, $n^2 = 2 \cdot (2k^2 + 2k) + 1$ which is an odd number – this is a contradiction because n^2 is assumed to be an even number. Therefore, n must be an even number. □

3 Answer to Problem 3

Some question may have sub-problems, and it will be useful to use the `enumerate` environment.

1. **Subtask (1)** My answer is 42.
2. **Subtask (2)** No, I would not do that.
3. **Subtask (3)** Yes, absolutely.

References

If you need to cite any sources, please do so here.

Recall that your answers must be in your own writing/typing (i.e., you should not copy-and-paste anything word-for-word).

1. In my answer to Problem 1, I used Theorem 5.x and Lemma 5.y in KT.
2. In my answer to Problem 2, I got an idea from this StackOverflow post: (provide a URL here).
3. In my answer to Problem 3-1, I learned the answer from the movie, “The Hitchhiker’s Guide to the Galaxy.”