# The domain of a composite function

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I have tried to find a systematic way of finding the domain of a composite function for a few months now. Today, I tried explaining this concept to a first-year maths student, and found myself horribly confused all over again. After some research in the library, I have finally found a definition of that can be used for this purpose (the internet mostly directs one to worked-through examples and no generalized definition, so I had to resort to the library):

$$D(f \circ g) = \{x | x \in D(g) \wedge g(x) \in D(f)\}$$

(Vaught, 1995:18)

Where $D(\lambda)$ = the domain of $\lambda$

## Example 1

Solve

$$D(\ln(\ln(\ln x)))$$

(This was in a MAM1000W past paper)

Solution:
Let $\ln(\ln(\ln x)) = f(g(h(x)))$

$$D(g \circ h) = \{x | x \in (0, \infty) \wedge \ln x \in (0, \infty)\}$$

Now, $\ln x \in (0, \infty) \Leftrightarrow x \in (1, \infty)$

$\therefore\ x \in (1, \infty)$

$$D(f \circ (g \circ h)) = \{x | x \in (1, \infty) \wedge \ln(\ln x) \in (0, \infty)\}$$

Now, $\ln(\ln x) \in (0, \infty) \Leftrightarrow \ln x \in (1, \infty) \Leftrightarrow x \in (e, \infty)$

$\therefore\ x \in (e, \infty)$

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## Example 2.1

Consider $f(x) = x + 1$ and $g(x) = x^2$ where $D(g) = [-2, 2]$.
Find $D(f \circ g)$

$$\begin{aligned} f \circ g(x) &= x^2 + 1 \\ D(f \circ g) &= \{x | x \in [-2, 2] \wedge x^2 \in \mathbb{R}\} \\ & x^2 \in \mathbb{R} \Leftrightarrow x \in \mathbb{R} \\ \therefore \ & x \in [-2, 2] \end{aligned}$$

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## Example 2.2

Consider the same example as Example 2.1, but with $D(f) = [-2, 1]$

$$\begin{aligned} D(f \circ g) &= \{x | x \in [-2, 2] \wedge x^2 \in [-2, 1]\} \\ & x^2 \in [-2, 1] \Leftrightarrow x^2 \in [0, 1] \Leftrightarrow x \in [-1, 1] \\ \therefore \ & x \in [-1, 1] \end{aligned}$$

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## References

Vaught, RL. 1995. Set theory: An introduction. 2nd edition. Boston: Birkhäuser.