# 1. Introduction

Welcome to your first lab assignment typed in LaTex!

## 1.1 Add several new commands!

In order to type the homework problem and its solution, why don't we first set several new commands?

First of all, by typing

```
\newtheorem{problem}[lemma]{Problem}
```

we can have like:

**Problem 1.1.** this is a problem
if we copy paste it several times, we have

**Problem 1.2.** this is another problem.

**Problem 1.3.** this is the third problem.
To write the solution, you may want to type

```
\renewcommand{\proofname}{\textbf{Solution:}}
```

Then, whenever you type

```
\begin{proof}
```

A solution.

```
\end{proof}
```

it will appear like

***Solution:*** A solution. □

# 2. Let's start writing our problems and solutions!

## 2.1 The first modeling problem!

**Problem 2.1.** (Set Covering Problem). AA Motor is trying to make cars for the next season, hence they need to buy enough raw materials: S tons of steel and P tons of plastic. Different suppliers, denoted by set D, have various offers for different combinations of raw materials: dealer d D sells $s_d$ tons of steel and $p_d$ tons of plastic for $c_d$ dollar. We know that AA Motor will not buy more than $W_d$ packages of raw material combination from any single supplier d D; moreover, because of the transportation budget, AA Motor will at most choose 5 suppliers to buy. Formulate a mathematical programming model to minimize the total cost.

***Solution:*** (Write your solution here.)
Sets :

D: set of suppliers

Parameters :

$$c_d \quad \text{cost of combo of steel and plastic that dealer d sells} \quad \forall d \in D$$

$$s_d \quad \text{tons of steel sold by dealer d} \quad \forall d \in D$$

$$p_d \quad \text{tons of plastic sold by dealer d} \quad \forall d \in D$$

$$W_d \quad \text{max number of packages of raw material combos from supplier d} \quad \forall d \in D$$

$$M \quad \text{max number of suppliers AA can buy from} = 5$$

$$S \quad \text{tons of steel to buy}$$

$$P \quad \text{tons of plastic to buy}$$

$$|K| \quad \text{some very large number}$$

Variables :

$$x_d \quad \text{number of packages bought from dealer d}$$

$$r_d \quad \text{Does AA Motor buy from dealer d?} \quad \forall d \in D (yes = 1, no = 0)$$

Objective function :

$$min \quad \sum_{d \in D} x_d \times c_d \quad \text{minimize the cost}$$

Constraints and Limitations :

$$x_d \leq W_d \quad \forall d \in D$$

$$\sum_{d \in D} y_d \leq M$$

$$\sum_{d \in D} s_d \times x_d \geq S$$

$$\sum_{d \in D} p_d \times x_d \geq P$$

$$y_d \geq \frac{\sum_{d \in D} x_d}{|K|}, \forall d \in D$$

$$x_d \geq 0$$

$$y_d \in \{1, 0\}$$

□

## 2.2 The second modeling problem!

**Problem 2.2.** (Knapsack Problem). You are going to climb Himalaya this summer, hence need to buy some supplies in order to survive there. First of all, you need to buy one backpack! Define the set of backpacks by K, and each k costs $c_k$ and has storage size $w_k$. Second, of course you have to bring some food! The set of food is denoted by F, and for each f  F, one unit costs $c_f$ , takes space $w_f$ in the backpack, and has calories $k_f$ . In order to survive in the wild, food you buy need to have total calories at least K. Safety supplies are also necessary: set of safety supplies is S, each one of s  S costs $c_s$, takes space $w_s$, and you need to buy $M_s$ different kinds of safety supplies. Please formulate a mathematical programming model to minimize the total cost.

***Solution:*** (Write your solution here.)
Sets :

K: set of backpacks

F: set of food

S: set of safety supplies

Parameters :

$$c_s \quad \text{cost of safety supply S} \quad \forall s \in S$$
$$c_f \quad \text{cost of food f} \quad \forall f \in F$$
$$c_k \quad \text{cost of backpack k} \quad \forall k \in K$$
$$w_s \quad \text{storage size of safety supply S} \quad \forall s \in S$$
$$w_f \quad \text{storage size of food F} \quad \forall f \in F$$
$$w_k \quad \text{storage size of backpack k} \quad \forall k \in K$$

Variables :

$$x_f \quad \text{number of food f to buy} \quad \forall f \in F$$
$$x_s \quad \text{number of safety supplies s to buy} \quad \forall s \in S$$
$$x_k \quad \text{number of k type backpacks to buy} \quad \forall k \in K$$

Objective function :

$$min \quad \sum_{s \in S} x_s \times c_s + \sum_{k \in K} x_k \times c_k + \sum_{f \in F} x_f \times c_f \qquad \text{minimize the cost}$$

Restrictions :

$$\sum_{f \in F} x_f \times w_f + \sum_{s \in S} x_s \times w_s \geq W_k \qquad \text{restricted storage size}$$
$$\sum_{f \in F} x_f \times k_f \geq k_n \qquad \text{calories from food meet the need}$$
$$x_s \geq M_s \quad \forall s \in S \qquad \text{adequate amount of safe equipment}$$

$$x_s, x_f, x_k \in \mathbb{N}_0 \quad \forall s \in S \quad \forall f \in F \quad \forall k \in K \qquad \text{decision variables are non-negative integers}$$

$\square$

## 2.3 The third modeling problem!

**Problem 2.3.** (Set Partitioning Problem). Formulate the following as a set partitioning problem. You are hosting a golf scramble at the country club you attend and have invited 11 of your friends and colleagues (12 total players each denoted by a letter A L). You need to divide the 12 players into 3 foursomes (3 groups with 4 players in each group). Because you know the personalities of the 12 players and how they will interact with each other you have determined that only the following groups would work out:

{ A,L,C,D } : 1

{ A,I,F,G } : 2

{ B,E,F,J } : 3

{ C,D,E,F } : 4

{ A,C,E,G } : 5

{ I,D,F,H } : -20

{ B,C,I,L } : 6

{ G,H,I,K } : -5

{ A,B,F,J } : 5

You want everyone to have fun and for there to be good competition so you want divide the players as fairly as possible. The value to the right of each group is a relative measure of how fair the group is (the higher the value, the more fair the group). Write a set partitioning formulation to solve this problem, maximizing the total fairness value. Be sure to clearly dene all notation.

***Solution:*** (Write your solution here.)
Sets:

$$\text{P: players}$$

$$\text{G: valid groups of players;} \forall g \in G, g \subseteq P$$

Parameters:

$$a_{pg} : \text{ whether player p is in group g (yes=1,no=0)}$$

$$v_g : \text{ value of group g, } \forall g \in G$$

Variables:

$x_g$ : decision of whether to include group g in your partitioning (yes=1, no=0),$\forall g \in G$

$$Max \quad \sum_{g \in G} x_g \times v_g$$

$$St \quad \sum_{g \in G} x_g \times a_{pg} = 1 \quad \forall p \in P$$

$$x_g \in \{0, 1\} \quad \forall g \in G$$

$\square$